DESIGN OF DUST FLOWMETERS

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We examine the motion of a flow of a dust–gas mixture in a dust flowmeter device. We derive an approximate formula for the calculation of the dust flow rate from the known pressure difference. Unlike [3], we take into consideration the change in the resistance factor for the particle over channel length.

The flow rate of pneumatically transported dust is frequently determined from the pressure difference across some throttling device (for example, across a Venturi tube [1, 2]). The calibration method usually employed for such devices is not always convenient in the case of large tube diameters and for great gas and dust flow rates. Approximate formulas are proposed in [3] for the design of dust flowmeters. The particle resistance factor is assumed to be constant, and independent of variations in the relative velocity of the particle over channel length. Below we derive a formula for the calculation of dust in such an installation, with consideration given to the continuous variation of the velocities for the solid and gaseous phases.

Let us examine the motion of a dust–gas mixture in a channel formed by converging conical and cylindrical surfaces (Fig. 1). We assume the gaseous phase to be incompressible. The temperature of the mixture along the length of the channel does not change. We neglect the interaction of the particles with the walls.

With motion through the conical portion of the tube, the particle is acted on by an aerodynamic force and the force of gravity. For small particles (with a diameter no larger than 0.1 mm) the action of the second force can be neglected, since the aerodynamic force is greater than the force of gravity by two orders of magnitude. If we assume that the increase in the kinetic energy of the particle is determined primarily by the increase in the horizontal velocity component, we can write the following equation of particle motion:

$$m_s \frac{dv}{dt} = -cf \frac{\rho_g w^2}{2}, \tag{1}$$

where w = u − v.

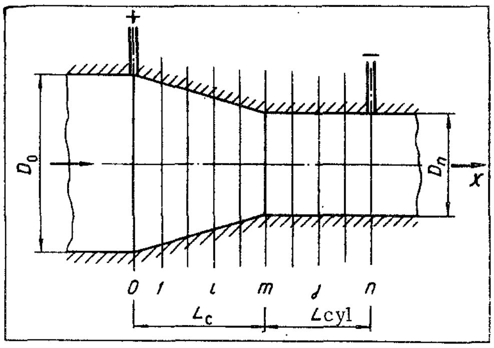

Fig. 1. Diagram of a dust flowmeter.

The solution of the differential equation (1) is substantially simplified if we divide the channel into small segments and assume that the velocity of the gaseous phase increases linearly within the limit of each segment:

$$u = u_{i-1} + k_i (x - x_{i-1}). \tag{2}$$

We denote the expression

$$A = cf \frac{\rho_g w}{2} \tag{3}$$

by A and we rewrite (1), with consideration of (2) and (3):

$$m_T \frac{vdv}{dx} + Av = A\left[u_{i-1} + k_i (x - x_{i-1})\right]. \tag{4}$$

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The resistance factor c in (3) is a function of velocity (of the Reynolds number) and of the shape factor for the particle [4, 5]:

$$c = k_{sh}(24\mathrm{Re}^{-1} + 4\mathrm{Re}^{-1/3}). \tag{5}$$

With consideration of (5), we can present (3) in the form

$$A = B(a + bw^{2/3}), \tag{6}$$

where

$$B = 0.3925k_{\Phi}d_s^2\rho_g;\quad a = 24\frac{\nu}{d_s};\quad b = 4\left(\frac{\nu}{d_s}\right)^{1/3}.$$

To obtain a solution for (4) in finite form, instead of the variable A let us introduce its mean integral value on the i-th segment, which is fully acceptable for a segment of limited length:

$$A_i = \frac{1}{w_i - w_{i-1}}\int_{w_{i-1}}^{w_i} A\,dw, \tag{7}$$

where i = 2, ..., m.

We know the value of w_{i-1} from the calculation of the previous segment; the value of w_i is determined by the method of successive approximations. By integration of (7) we obtain

$$A_i = B\left(a + \frac{3}{5}b\,\frac{w_i^{5/3} - w_{i-1}^{5/3}}{w_i - w_{i-1}}\right). \tag{8}$$

With consideration of the above, the solution for (4) has the form

$$v_i = D_i(\lambda_i + 1 - 2y_i)^{\frac{1-\lambda_i}{2\lambda_i}}(\lambda_i - 1 + 2y_i)^{-\frac{1+\lambda_i}{2\lambda_i}}, \tag{9}$$

where

$$y_i = \frac{u_i}{v_i};\quad \lambda_i = \sqrt{1 + \frac{4m_sk_i}{A_i}};$$

$$D_i = v_{i-1}(\lambda_i + 1 - 2y_{i-1})^{\frac{\lambda_i-1}{2\lambda_i}}(\lambda_i - 1 + 2y_{i-1})^{\frac{\lambda_i+1}{2\lambda_i}}.$$

Having successively determined v_i in all segments, we find the particle velocity v_m at the end of the converging section.

Let us now examine the motion of the particle in the cylindrical portion of the dust flowmeter, and this motion is also described by (1). For the solution of (1) we will divide the length of the cylindrical portion into n−m segments and we will find the mean integral value of A on the j-th segment. As a result, for the j-th segment, with consideration of the fact that u = const, we have

$$\gamma_j\left[\ln\frac{(y_{j-1}-1)\,y_j}{(y_j-1)\,y_{j-1}} - \frac{y_{j-1}-y_j}{y_{j-1}y_j}\right] = x_j - x_{j-1}, \tag{10}$$

where

$$\gamma_j = \frac{m_sv_m}{A_j},\quad y_j = \frac{v_m}{v_j}.$$

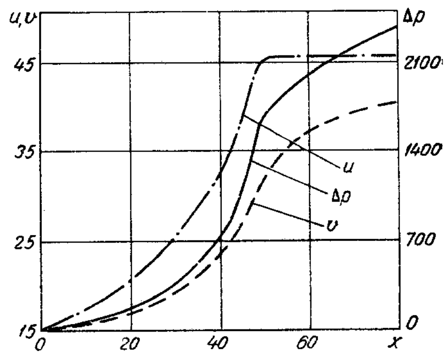

Fig. 2. Variation in the parameters u (m/sec), v (m/sec), and Δp (n/m^2) of a two-phase flow along the length x (mm) of the channel of a dust flowmeter.

Having successively determined the particle velocity (v_j) in all sections of the cylindrical portion, we will find the particle velocity v_n at the point at which the negative-pressure orifice of the dust flowmeter is located.

Let us proceed to the determination of the pressure difference. The pressure drop across a channel of variable cross section is a result of the increase in the kinetic energy of the gas and of the particles. For an infinitely small segment dx we write [7]

$$\rho_g u\,du + \mu\rho_g u\,dv = -dp. \tag{11}$$

After integration of (11) for the i-th segment we obtain the following expression:

$$\Delta p_i = \frac{\rho_g}{2}(u_i^2 - u_{i-1}^2) + \frac{\mu\rho_g}{2}\left(u_i v_i - u_{i-1}v_{i-1} - \int_{y_{i-1}}^{y_i} v^2 dy\right). \tag{12}$$

The integral $I_i = \int_{y_{i-1}}^{y_i} v^2 dy$ after substitution of (9) is brought to the form

$$I_i = E_i\left[\int_0^{t_i} t^{p_i'-1}(1-t)^{q_i'-1}dt - \int_0^{t_{i-1}} t^{p_i'-1}(1-t)^{q_i'-1}dt\right]. \tag{13}$$

The expressions in the brackets represent the beta function [6]

$$B_{t_i}(p_i' q_i') = \int_0^{t_i} t^{p_i'-1}(1-t)^{q_i'-1}dt = \frac{t_i^{p_i'}}{p_i'}\,{}_2F_1(p_i',\ 1-q_i',\ 1+p_i',\ t_i), \tag{14}$$

where ${}_2F_1(p_i',\ 1-q_i',\ 1+p_i',\ t_i)$ is a hypergeometric function presented in the form of a series. With accuracy sufficient for practical purposes, we can limit ourselves exclusively to the first three terms of the series

$${}_2F_1(\alpha,\ \beta,\ \gamma,\ z) = 1 + \frac{\alpha\beta}{\gamma\cdot 1}z + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)\cdot 1\cdot 2}z^2 + \dots \tag{15}$$

We have introduced the following notation into (13)-(15):

$$E_i = -\frac{D_i^2}{4\lambda_i},\quad p_i' = \frac{1}{\lambda_i},\quad q_i' = -\frac{1}{\lambda_i},\quad t_i = \frac{\lambda_i + 1 - 2y_i}{2\lambda_i},$$
$$\alpha = p_i',\ \beta = 1 - q_i',\ \gamma = 1 + p_i',\ z = t_i.$$

After having calculated the pressure drop across each segment we find the pressure drop in the converging portion:

$$\Delta p_c = \sum_i \Delta p_i. \tag{16}$$

The pressure drop across the cylindrical segment, a consequence of the longitudinal acceleration of the particles, is determined from the following formula, taken from [7]:

$$\Delta p_{cyl} = \mu\rho_g u_m(v_n - v_m). \tag{17}$$

The total pressure drop, established instrumentally, is written in the form

$$\Delta p = (1+\zeta)\frac{\rho_g}{2}(u_m^2 - u_0^2) + \mu\rho_g u_m(v_n - v_m) + \frac{\mu\rho_g}{2}\sum_i(u_i v_i - u_{i-1}v_{i-1} - I_i). \tag{18}$$

Here we have introduced the local gas-friction factor ζ for the conical portion.

Given a known Δp, from (18) we find the flow rate for the solid phase:

$$G_s = G_g\frac{\Delta p - (1+\zeta)\frac{\rho_g}{2}(u_m^2 - u_0^2)}{\rho_g\left[\frac{1}{2}\sum_i(u_i v_i - u_{i-1}v_{i-1} - I_i) + u_m(v_n - v_m)\right]}. \tag{19}$$

As an example, let us consider the case of the motion of a two-phase air–coal dust mixture (particle size 20 μ). The original data are: D_0 = 70 mm; D_n = 40 mm; L_c = 50 mm; L_{cyl} = 30 mm; μ = 1 kg/kg; ρ_g = 1.2 kg/m^3; $\nu = 1.5\cdot 10^{-5}$ m^2/sec; and $u_0 = v_0$ = 15 m/sec. Figure 2 shows the change in the phase velocities and in the pressure difference along the length.

The results from the calculation of the dust flow rate in accordance with (19) for specified flow and channel parameters differ from the values derived without consideration of the 15% variation in the particle resistance factor [3].

NOTATION

m_s is the mass of the solid particle;
v is the velocity of the solid particle;
u is the velocity of the gas phase;
t is the time;
c is the particle resistance factor;
f is the lateral cross-sectional area of the particle;
ρ_g is the density of the gas phase;
w is the relative particle velocity;
k_i is the coefficient of linear velocity increase for the gas phase;
x is the coordinate along the channel axis;
k_{sh} is the particle shape factor;
Re is the Reynolds number;
d_s is the particle diameter;
ν is the coefficient of kinematic viscosity;
i is the sequential number for the cross sections in the converging portion of the dust flowmeter;
j is the sequential number for the cross sections in the cylindrical portions;
μ is the coefficient of flow concentration;
p is the pressure of the gas phase in the flow;
G_g is the mass flow rate of the gas phase;
G_s is the mass flow rate of the solid phase;
ζ is the local friction factor.

Subscripts

0 is the flow parameter at the inlet to the channel of the dust flowmeter;
m is the parameter at the end of the converging portion;
n is the parameter in the cross section of the negative-pressure orifice;
g is the gas phase;
s is the solid phase;
c is the converging portion;
cyl is the cylindrical portion.

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